

NAG Toolbox for MATLAB

f07ug

1 Purpose

f07ug estimates the condition number of a real triangular matrix, using packed storage.

2 Syntax

```
[rcond, info] = f07ug(norm_p, uplo, diag, n, ap)
```

3 Description

f07ug estimates the condition number of a real triangular matrix A , in either the 1-norm or the ∞ -norm, using packed storage:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^T)$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function computes $\|A\|_1$ or $\|A\|_\infty$ exactly, and uses Higham's implementation of Hager's method (see Higham 1988) to estimate $\|A^{-1}\|_1$ or $\|A^{-1}\|_\infty$.

4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **norm_p** – string

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

norm_p = '1' or 'O'

$\kappa_1(A)$ is estimated.

norm_p = 'I'

$\kappa_\infty(A)$ is estimated.

Constraint: **norm_p** = '1', 'O' or 'I'.

2: **uplo** – string

Indicates whether A is upper or lower triangular.

uplo = 'U'

A is upper triangular.

uplo = 'L'

A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

3: **diag** – string

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'

A is a nonunit triangular matrix.

diag = 'U'

A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **n** – int32 scalar

n , the order of the matrix A .

Constraint: $n \geq 0$.

5: **ap**(*) – double array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The n by n triangular matrix A , packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in **ap**($i + j(j - 1)/2$) for $i \leq j$;

if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in **ap**($i + (2n - j)(j - 1)/2$) for $i \geq j$.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced; the same storage scheme is used whether **diag** = 'N' or 'U'.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

work, iwork

5.4 Output Parameters

1: **rcond** – double scalar

An estimate of the reciprocal of the condition number of A . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, A is singular to working precision.

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **norm_p**, 2: **uplo**, 3: **diag**, 4: **n**, 5: **ap**, 6: **rcond**, 7: **work**, 8: **iwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to f07ug involves solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately n^2 floating-point operations but takes considerably longer than a call to f07ue with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this function is f07uu.

9 Example

```
norm_p = '1';
uplo = 'L';
diag = 'N';
n = int32(4);
ap = [4.3;
      -3.96;
       0.4;
      -0.27;
      -4.87;
       0.31;
      0.070000000000000001;
      -8.02;
      -5.95;
       0.12];
[rcond, info] = f07ug(norm_p, uplo, diag, n, ap)

rcond =
    0.0086
info =
     0
```